Multi-dimensional FE models for multifield analyses of composite realistic helicopter blades

M. Filippi*, E. Carrera†

Mul[2] Group
Department of Mechanical and Aerospace Engineering, Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino, Italy.

1 Variable kinematic CUF models

Carrera Unified formulation for uni-dimensional elements (1D) expresses the displacement field $\mathbf{u}^T = [u_x\, u_y\, u_z]$ as an arbitrary number of products between cross-sectional functions $F_\tau(x,z)$ and the generalized displacements $u_\tau(y,t)$ that depend on the longitudinal coordinate and time. If the finite element method is adopted, the generalized displacements are approximated using the shape functions, $N_i(y)$, and the vector of nodal displacement $\mathbf{q}_{\tau i}(t)$. Therefore the displacement field is being approximated as:

$$\mathbf{u} = F_\tau(x,z) N_i \mathbf{q}_{\tau i}(t) \quad \tau = 1, ..., M \quad i = 1, ..., N_{nodes} \tag{1}$$

the number of elements nodes are N_{nodes}, τ is related to the expansion used for defining the cross-sectional kinematics and its maximum value, M, is an input parameter of the analysis. Moreover the parameter M defines the polynomial order for the Taylor-type models, named TEM. In case of Lagrange-type CUF models (LE) the beam kinematics is obtained as combination of Lagrange polynomials that are defined within sub-regions (or elements) delimited by arbitrary numbers of points (or nodes). The governing equilibrium equations of the beam are derived via the Principle of Virtual Displacements (PVD):

$$\delta L_{int} = \int_V \delta\epsilon^{\mathbf{T}} \sigma dV = \delta L_e \tag{2}$$

for a static thermo-elastic analysis δL_e is considered equal to zero. The Hooke law for a linear thermoelastic material is:

$$\sigma = \mathbf{C}\epsilon^e \tag{3}$$

where $\mathbf{C}$ is the forth order tensor of elastic moduli. In the linear thermoelasticity, the elastic strain vector ϵ^e is equal to

$$\epsilon^e = \epsilon - \epsilon^t \tag{4}$$

where ϵ denotes the total strain vector and ϵ^t is the strain vector caused by the temperature change $\Delta T = T - T_0$, that is

$$\epsilon^t = \alpha(\Delta T) \tag{5}$$

where T_0 is the reference temperature. The steady-state temperature distribution T may be, in general, a function of all three space coordinates. The vector α stands for the vector of linear thermal expansion coefficients. Moreover, the linear strain-displacement relations can be written as

$$\epsilon = \mathbf{bu} \tag{6}$$

Thus, substituting Eq. 3 and 4 into Eq. 2, the internal work becomes

$$\begin{aligned} \delta L_{int} = & \int_V \delta\epsilon^{\mathbf{T}}\sigma dV = \int_V \delta\epsilon^{\mathbf{T}}(C\epsilon - \beta\Delta T)dV = \int_V \delta\mathbf{u}^{\mathbf{T}}\mathbf{b}^{\mathbf{T}}(C\mathbf{bu} - \beta\Delta T)dV = \\ & = \underbrace{\int_V \delta\mathbf{u}^{\mathbf{T}}\mathbf{b}^{\mathbf{T}}(C\mathbf{bu})dV}_{\mathbf{K}_{\mathbf{ST}}^{\mathbf{ij}\tau s}} - \underbrace{\int_V \delta\mathbf{u}^{\mathbf{T}}\mathbf{b}^{\mathbf{T}}\beta\Delta T dV}_{\mathbf{F}_{\mathbf{T}}^{\mathbf{js}}} \end{aligned} \tag{7}$$

with $\beta = \mathbf{C}\alpha$. $\mathbf{F}_{\mathbf{T}}^{\mathbf{js}}$ is the vector of thermal loads generated by an already known temperature profile.

*Assistant Professor. Corresponding author. E-mail: matteo.filippi@polito.it
†Professor of Aerospace Structures and Aeroelasticity

2 Numerical results

The one-dimensional model has been used to investigate the static and the dynamic response of a realistic composite rotor blade with a realistic airfoil. The cross-section has been divided into 101 sub-domains corresponding to the number of Lagrange elements with either four or nine nodes. A constant temperature distribution was assumed, and this effect on the natural frequencies of the structure was analysed.

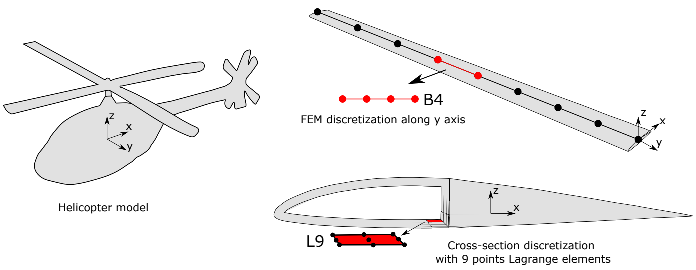

Figure 1: Geometry and section details of the blade with realistic airfoil

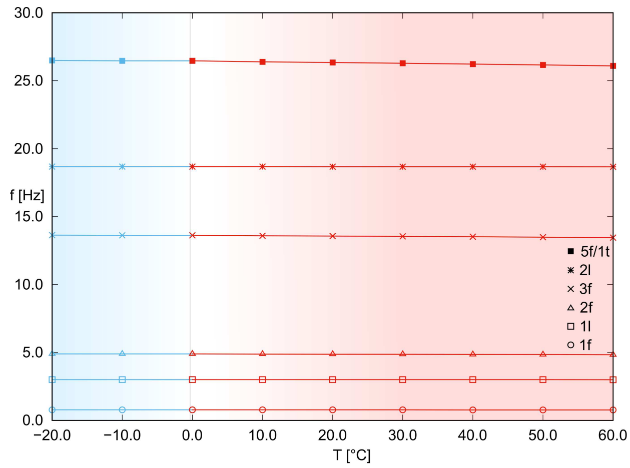

Figure 2: Natural frequencies behaviour of the isotropic blade with temperature increasing.

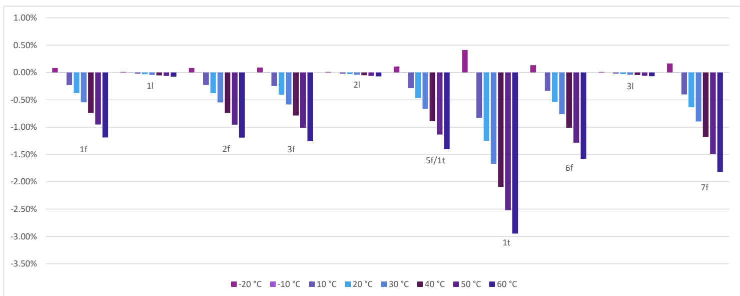

Figure 3: Discrepancies on natural frequencies behaviour $\left(\frac{f - f_{\Delta T=0}}{f_{\Delta T=0}}\right)$ of the realistic blade with temperature increasing. (*f: flapwise shape mode, l: chordwise shape mode, t: torsional shape mode.*)

Figure 2 shows the variations along with the temperature change (ΔT) of the first six natural frequencies. The discrepancies with respect to the case in which ΔT is equal to zero are shown in Fig 3. For the blade with a realistic airfoil, the most remarkable differences with increasing temperature occur for the first torsional frequency. As with the rectangular profile isotropic blade, the increase in temperature does not affect the frequency values of the chordwise shape modes.